

Return Optimization Notes, Principal Protection Notes, and Other Remarkable Structured Investment Vehicles

Donald Richards
Penn State University

Return Optimization Notes

As usual, we begin with a news article

A small investor who bought strangely-named securities

She lost her total investment in a \$225,000 “guaranteed principal protected note” and a \$75,000 “reverse optimization note”

Her lawyer argued that the notes were unsuitable “for unsophisticated investors”

Jennifer Papini, “[Investor wins Lehman note arbitration](#),” Wall Street Journal, December 5, 2009

“There are ... thousands more cases” like this one

The word “optimization” here is a very bad sign

“Optimization” is common to mathematics, but not to investing

I sense that financial engineers have been at work here: Uh oh!

Search the Internet for the phrase “return optimization”, and sure enough ...

Markowitz (1956) “developed mathematical methods for solving the risk-return optimization problem”

My conjecture

Motivated by Markowitz's paper, some smart-ass financial engineers devised "Risk-Return Optimization Notes"

Then the salespeople said: "The word 'Risk' will scare the customers!"

The legal department said: "And 'Risk' will increase our legal liability when these things crash!"

So the engineers said, "Okay; remove 'Risk' and call them 'Return Optimization Notes!' We couldn't care less!"

And down went the investors with their negative-return optimization notes

What is a “return optimization note”?

SEC website: Read a prospectus for a “Return Optimization Note with Partial Protection Linked to the S&P500 Index”

Price: \$10 per note (as an example, we suppose that we buy 1 note)

Maturity period: 369 days

Why 369? Why not 365 or 366?

I_0 : The S&P500 index at the start of the maturity period

I_1 : The S&P500 index at the end of the maturity period

The percentage return on the index: $I = \frac{I_1 - I_0}{I_0}$

Note that I can be negative

The payment at maturity depends on a mysterious number, M

It was unclear how M is computed; however, it is stated that

$$0.25 \leq M \leq 0.30$$

Your net payment, i.e., the return on your \$10 capital, is

$$\text{Net payment} = \begin{cases} \min(50I, 10M), & \text{if } I \geq 0 \\ 10I, & \text{if } I < 0 \end{cases}$$

This is an interesting formula

Suppose the S&P500 declines 42% during the 369-day period

Then $I = -0.42$ and your net payment is -4.2

You lose 42% of your capital in 369 days

Data from Google Finance

Oct. 05, 2007: S&P500 closed at 1557.59

Oct. 10, 2008: S&P500 closed at 899.22

$$I = \frac{899.22 - 1557.59}{1557.59} = -0.42$$

A 42% decline in the S&P500 in one year

Return optimization notes may also be unsuitable for sophisticated investors!

Suppose the S&P500 had advanced 42%

The net payment is bounded above by $10M$ if $I > 0$

If $I = +.42$ then the net payment is at most $10M$

Recall that $0.25 \leq M \leq 0.30$

If the S&P500 advances 42% then your return is at most 30%

Why risk losing 100% for a small chance to make at most 30% ?

Denote by R the percentage net payment on your \$10 capital

We are probabilists

We want the probability distribution, mean, and variance, of R

$$R = \frac{\text{Net payment}}{10} = \begin{cases} \min(5I, M), & \text{if } I \geq 0 \\ I, & \text{if } I < 0 \end{cases}$$

The distribution of R is determined by the distribution of I

Elementary undergraduate-level calculations

Observe that

$$R = \begin{cases} M, & \text{if } I > M/5 \\ 5I, & \text{if } 0 \leq I \leq M/5 \\ I, & \text{if } I < 0 \end{cases}$$

Apply the Rule of Weighted Averages (Law of Total Probability)

This gives us the expected value of R :

$$\begin{aligned} E(R) &= M \cdot P(I > M/5) \\ &\quad + 5 E(I \mid 0 \leq I \leq M/5) \cdot P(0 \leq I \leq M/5) \\ &\quad + E(I \mid I < 0) \cdot P(I < 0) \end{aligned}$$

What does this formula tell us?

First, some historical remarks

Return optimization notes were invented in 2006 when the S&P500 was near record levels

So, R was likely to become negative

The S&P500 was unlikely to go upwards for 369 days more

Note: In my [2006 Bowling Green lectures](#), I argued that markets were due for sharp declines

Conclusion: $P(I \geq 0) \simeq 0$

Return to the formula for $E(R)$: Because

$$E(I \mid I < 0) \leq 0$$

and

$$E(I \mid 0 \leq I \leq M/5) \leq M/5$$

then

$$\begin{aligned} E(R) &\leq M \cdot P(I > M/5) + 5 \cdot (M/5) \cdot P(0 \leq I \leq M/5) \\ &= M \cdot [P(I > M/5) + P(0 \leq I \leq M/5)] \\ &= M \cdot P(I \geq 0) \\ &\simeq 0 \end{aligned}$$

The average buyer of return optimization notes in 2007 could not expect to have positive net return

Suppose that $P(I \geq 0) = 10\%$; then, $P(I < 0) = 90\%$

We again apply the Rule of Weighted Averages

$$\begin{aligned} E(R) &\simeq E(I \mid I < 0) \cdot P(I < 0) \\ &= (-0.42) \cdot (0.90) \\ &= -0.378 \end{aligned}$$

I would have estimated in 2007 that return optimization notes will generate average losses of at least **38%** to investors

Conclusions on return optimization notes

We used simple bounds to obtain $E(R) \leq 0$

Using data (from Google Finance) on the S&P500, we can estimate accurately all terms in the formula for $E(R)$

The conclusion: $E(R) \ll 0$, so the average buyer was doomed to suffer massive losses

It was ominous that the notes limited profits to 30% but did not limit losses

This allowed the bankers to make money if the markets continued upwards

Return optimization notes benefited large stockholders wanting to insure against large declines in the stock market

As investors bought return optimization securities, they insured large stockholders who then were able to avoid direct stock sales that might have triggered widespread market declines

Ultimately, the sellers of return optimization notes were hedging their risk at the expense of the buyers

Why would an investment adviser, in exercising fiduciary care of clients' funds, advise small investors to buy these notes?

Yield Magnet Notes

Search the Internet with keywords: yield magnet notes prospectus supplement

Issue Price: \$1,000

Redemption Price: \$1,000

The notes are based on a basket of 15 stocks from the Dow Jones “Global Titans Index”:

- | | |
|-----------------------|------------------------|
| 1. AIG | ⋮ |
| 2. BP | 13. Toyota |
| 3. Cisco Systems Inc. | 14. Vodafone Group plc |
| ⋮ | 15. Wal-Mart |

Settlement Date: 3/15/06

Redemption Date: 3/15/11

Interest will be paid annually on 3/15 (the "payment" date)

How is the coupon rate calculated? An interesting procedure

Determination date (D-date): Three business days before 3/15

Period	Coupon Rate
3/15/06-3/15/07	5.5% fixed rate ("the bait")
3/15/07-3/15/11	Variable coupon rate

3/15/07-3/15/08: The coupon rate is computed as follows:

Step 1: For stocks $i = 1, \dots, 15$, calculate

$$\delta_i = \frac{\text{Stock price on D-date} - \text{Stock price on 3/15/07}}{\text{Stock price on 3/15/07}}$$

Step 2: Calculate

$$\theta_i = \begin{cases} -0.125, & \text{if } \delta_i < -0.125 \\ \delta_i, & \text{if } -0.125 \leq \delta_i \leq 0.08 \\ 0.08, & \text{if } \delta_i > 0.08 \end{cases}$$

Note: An upper limit of 8% on θ_i vs. a lower limit of -12.5%

Step 3: Calculate $\bar{\theta} = \frac{1}{15} \sum_{i=1}^{15} \theta_i$

Step 4: For the period 3/15/07-3/15/08,

$$\text{Coupon rate} = \begin{cases} 0, & \text{if } \bar{\theta} < 0 \\ \bar{\theta}, & \text{if } 0 \leq \bar{\theta} \leq 0.08 \\ 0.08, & \text{if } \bar{\theta} > 0.08 \end{cases}$$

The coupon rate for 3/15/08-3/15/11 will be no smaller than the rate paid for 3/15/07-3/15/08

If $\theta_i = 0.08$ on any D-date then θ_i is kept at 0.08 for all later D-dates ("Yield Magnet")

Concerns about Yield Magnet Notes

The client does not know the coupon rate until the D-date, which is close to the interest payment rate

A small retired investor will have a hard time with the calculation of the coupon rate

An upper limit of 8% on θ_i vs. a lower limit of -12.5%

The banker benefits more than the client if stocks do well

The client does worse than the banker if stocks do poorly

Income tax woes? Prospectus: “comparable yield may be more than actual yield”, “contingent payment debt instrument”

The fixed coupon rate, 5.5%, for Year 1: A “bait” to hook the fish?

The probabilistic behavior of the coupon rate is complicated

How do we estimate the probability that $\bar{\theta} > 0$?

The δ_i are correlated random variables, as are the θ_i

These notes were sold to investors as stock markets neared record highs

Has the coupon rate has been negative since fall, 2008?

Reverse Exchangeable Securities

These are based on a single stock in a list chosen by the investment banker

Uh oh ... is that list chosen at random?

Or is it a list of stocks in which they already have huge holdings and want to hedge their risk?

Price: \$1,000

Maturity date: 1 year (for example)

Coupon rate: 12%, paid in quarterly installments

How is the noteholder repaid at maturity?

If the stock has advanced: \$1,000 in cash

If the stock has declined: Shares that were priced at \$1,000 at the start of the holding period

The total return on capital:

$$\text{Return} = \begin{cases} 12\%, & \text{if stock rises} \\ -r\%, & \text{if stock falls } r\% \end{cases}$$

It is easy to apply probability theory to these securities

Rule of Weighted Averages

$$E(\text{Return}) = 0.12 \cdot P(\text{Stock rises}) \\ - (r/100) \cdot P(\text{Stock falls } r\%)$$

Suppose $P(\text{Stock rises}) \approx 0$, then

$$E(\text{Return}) \approx -(r/100) \cdot P(\text{Stock falls } r\%) < 0$$

If the notes are sold when stocks are near record heights then the average buyer cannot expect to have positive net return

Concerns about Reverse Exchangeable Notes

Created as stocks neared record heights, $P(\text{Stock rises}) \approx 0$

Ideal for investment bankers who want a ready market for stock that they may need to unload quickly

Or they need to hedge call options on the stock

If the stock advances sharply then they pay the noteholder 12% and retain any further increase

If the stock declines then they put the shares to the noteholder

As noteholders rush to sell the stock to limit their losses, they'll depress the stock price more and have even larger losses

The bankers are also sure to short the stock when they see 10^4 sell orders (It's not personal, it's just business)

The coupon rate, 12%, was far higher than the Treasury rate

Such a coupon rate suggests junk-bond status

Homework: Determine what ratings were assigned by the ratings agencies to reverse exchangeable notes?

We infer that the financial engineers who devised these notes were expecting the stock market to decline by far more than 12%

They were correct in that expectation

Other bizarre securities designed by financial engineers

Guaranteed Principal-Protected Notes

“Guaranteed”? By whom?

“Principal-protected”? By whom?

Principal Protected Absolute Return Barrier Notes

“Absolute return barrier”?

“Ask not for whom the bell tolls ...”

Guaranteed Principal-Protected Notes With Partial Exposure to the S&P500

Guaranteed Principal-Protected Notes With Partial Exposure to **Commodities!**

These securities have wiped out many small investors, leading to huge numbers of lawsuits

Do financial engineers, especially those in academia, bear any responsibility for these events?

The grief suffered by small investors who bought these securities

Gretchen Morgenson, “Fair Game: ‘100% Protected’ Isn’t as Safe as It Sounds”, New York Times, March 23, 2010

The article ends on a superbly incisive note:

Gretchen Morgenson, “Fair Game: ‘100% Protected’ Isn’t as Safe as It Sounds”, New York Times, March 23, 2010

The article ends on a superbly incisive note:

“Add these securities to the growing pile of Wall Street inventions that benefit ...

Gretchen Morgenson, “Fair Game: ‘100% Protected’ Isn’t as Safe as It Sounds”, New York Times, March 23, 2010

The article ends on a superbly incisive note:

“Add these securities to the growing pile of Wall Street inventions that benefit ...

“wait for it,

Gretchen Morgenson, “Fair Game: ‘100% Protected’ Isn’t as Safe as It Sounds”, New York Times, March 23, 2010

The article ends on a superbly incisive note:

“Add these securities to the growing pile of Wall Street inventions that benefit ...

“wait for it,

“wait for it ...

Gretchen Morgenson, “Fair Game: ‘100% Protected’ Isn’t as Safe as It Sounds”, New York Times, March 23, 2010

The article ends on a superbly incisive note:

“Add these securities to the growing pile of Wall Street inventions that benefit ...

“wait for it,

“wait for it ...

“Wall Street.”

Why would an investment advisor, in exercising fiduciary duty to clients, recommend that small investors purchase these notes?

Seth Klarman: [Legendary Investor Is More Worried Than Ever](#)

By keeping interest rates low, the federal gov't is “giving bad advice” to investors to swap their safe, 0%-interest bank accounts for unsafe securities having higher risk than perceived

My bet: The current financial crisis will last until at least 2018

Would you like to bet me? Call me ...

These examples raise the questions:

What is an “investment”?

Who is an “investor”?

We will address these questions in the next two lectures